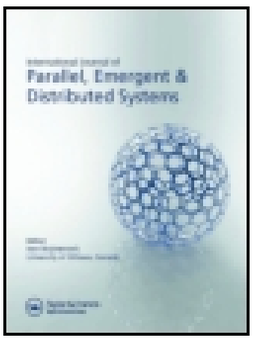

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Hidden and self-excited attractors in Chua circuit: synchronization and SPICE simulation

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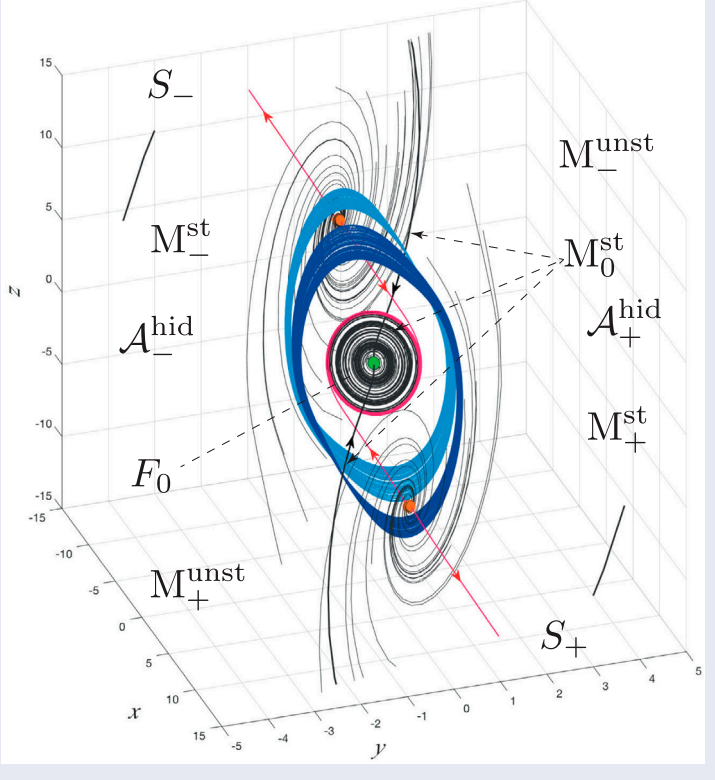
ABSTRACT

Nowadays various chaotic secure communication systems based on synchronization of chaotic circuits are widely studied. To achieve synchronization, the control signal proportional to the difference between the circuits signals, adjust the state of one circuit. In this paper the synchronization of two Chua circuits is simulated in Simulation Programs with Integrated Circuit Emphasis. It is shown that the choice of control signal is not straightforward, especially in the case of multistability and hidden attractors.



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Two symmetric hidden chaotic attractors (blue), trajectories (red) from unstable manifolds of two saddle points are either attracted to locally stable zero equilibrium, or tend to infinity; trajectories (black) from stable manifolds tend to equilibria.

1. Introduction: hidden and self-excited attractors in Chua circuit

Since the first chaotic behavior in dynamical systems was revealed by numerical integration [1], the researchers started to be interested in circuit implementation of chaos (see, e.g. [2,3] and others), which allows one to ensure that the pseudo-orbits can be traceable by actual orbits (see, e.g. the corresponding discussion on shadowing in [4,5]). At the same time various engineering perspectives of chaotic circuits application have been found [6].

The Chua circuit, invented in 1983 by Leon Chua [7,8], is the simplest electronic circuit exhibiting chaos. Consider one of classical Chua circuits shown in Figure 1.

The circuit consists of passive resistors (R and R_0), capacitors (C_1 and C_2), conductor L, and one nonlinear element with characteristics $f(\cdot)$, called Chua diode. It is described by the following equations

$$\begin{aligned}
\frac{dv_1}{dt} &= \frac{1}{C_1}\left(\frac{1}{R}(v_2 - v_1) - f(v_1)\right),\\
\frac{dv_2}{dt} &= \frac{1}{C_2}\left(\frac{1}{R}(v_1 - v_2) + i_L\right),\\
\frac{di_L}{dt} &= -\frac{1}{L}(v_2 + R_0 i_L),\\
f(v_1) &= \begin{cases} G_b v_1 + (G_b - G_a)E_1, & \text{if } v_1 \le -E_1,\\ G_a v_1, & \text{if } |v_1| < E_1,\\ G_b v_1 + (G_a - G_b)E_1, & \text{if } v_1 \ge E_1. \end{cases}
\end{aligned} \tag{1}$$

Here v_1 and v_2 are voltages across capacitors C_1 and C_2, respectively, i_L is a current through conductor L, function $f(v_1)$ is volt-ampere characteristics of Chua's diode. In the following discussion we choose C_2 such that $RC_2 = 1$ and put $E_1 = 1$. By the introduction of new variables

$$\begin{aligned}
x = v_1,\ y = v_2,\ z = Ri_L,\\
\alpha = \frac{1}{RC_1}, \beta = \frac{R}{L}, \gamma = \frac{R_0}{L}\\
\psi(x) = Rf(x),\ RG_a = m_0,\ RG_b = m_1,
\end{aligned} \tag{2}$$

system (1) is transformed to the following form

$$\begin{aligned}
\frac{dx}{dt} &= \alpha(y - x - \psi(x)),\\
\frac{dy}{dt} &= x - y + z,\\
\frac{dz}{dt} &= -(\beta y + \gamma z),\\
\psi(x) &= m_1 x + (m_0 - m_1)\mathrm{sat}(x),\\
\mathrm{sat}(x) &= \begin{cases} -1, & \text{if } x \le -1,\\ x, & \text{if } |x| < 1,\\ 1, & \text{if } x \ge 1. \end{cases}
\end{aligned} \tag{3}$$

Until recently there had been found Chua attractors, which are excited from unstable equilibria only and, thus, can be easily computed (see, e.g. a gallery of Chua attractors in [9]). Note that L. Chua

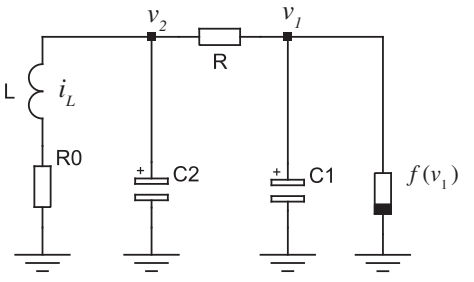

Figure 1. Chua circuit.

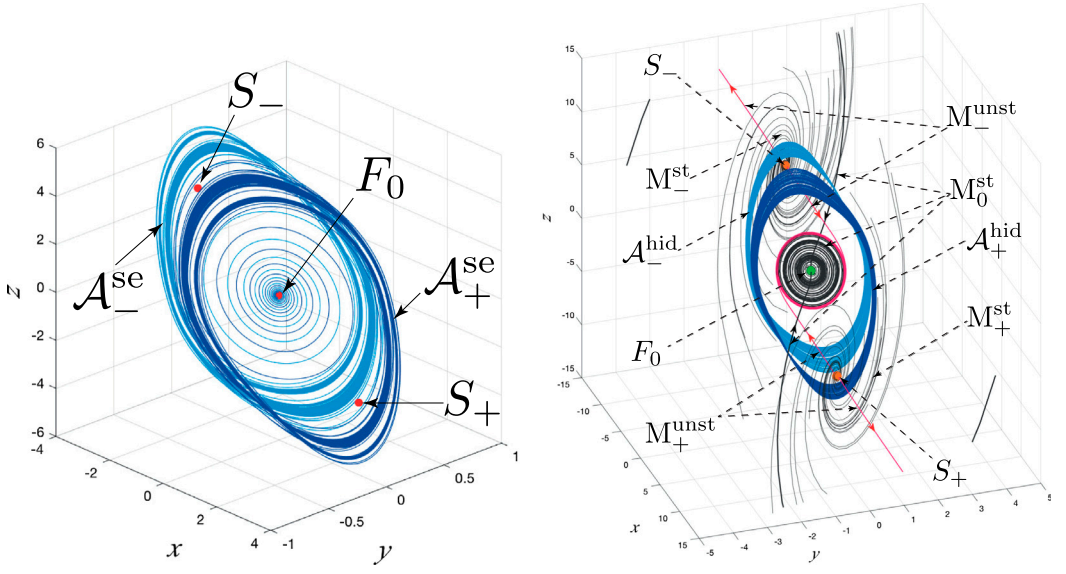

Figure 2. Left subfigure: two symmetric self-excited attractors (blue) excited from zero unstable equilibrium (parameters $\alpha = 15$, $\beta = 28$, $\gamma = 0$, $m_0 = -5/7$, $m_1 = -8/7$). Right subfigure: two symmetric hidden chaotic attractors (blue), trajectories (red) from unstable manifolds $\mathrm{M}^{\rm unst}_{\pm}$ of two saddle points $S_{\pm}$ are either attracted to locally stable zero equilibrium F_0, or tend to infinity; trajectories (black) from stable manifolds $\mathrm{M}^{\rm st}_{0,\pm}$ tend to F_0 or $S_{\pm}$. (parameters $\alpha = 8.4562$, $\beta = 12.0732$, $\gamma = 0.0052$, $m_0 = -0.1768, m_1 = -1.1468$)

[8], analyzing various cases of attractors in Chua's circuit, did not admit the existence of attractors of another type — so called *hidden attractors*, being discovered later in his circuits. An attractor is called a *self-excited attractor* if its basin of attraction intersects an arbitrarily small open neighborhood of equilibrium, otherwise it is called a *hidden attractor* [10–13]. Hidden attractor has basin of attraction which does not overlap with an arbitrarily small vicinity of equilibria.

For example, hidden attractors are attractors in systems without equilibria or with only one stable equilibrium (a special case of multistability and coexistence of attractors). The *hidden vs self-excited classification of attractors* was introduced in connection with the discovery of the first hidden Chua attractor [10,14–17]. The *Leonov–Kuznetsov's classification of attractors as hidden or self-excited* is captured much attention of scientists from around the world and hidden Chua attractors have become intensively studied (see, e.g. [18–32]. Examples of self-excited and hidden Chua attractors visualized by numerical integration of system (3) in MATLAB are shown in Figure 2.

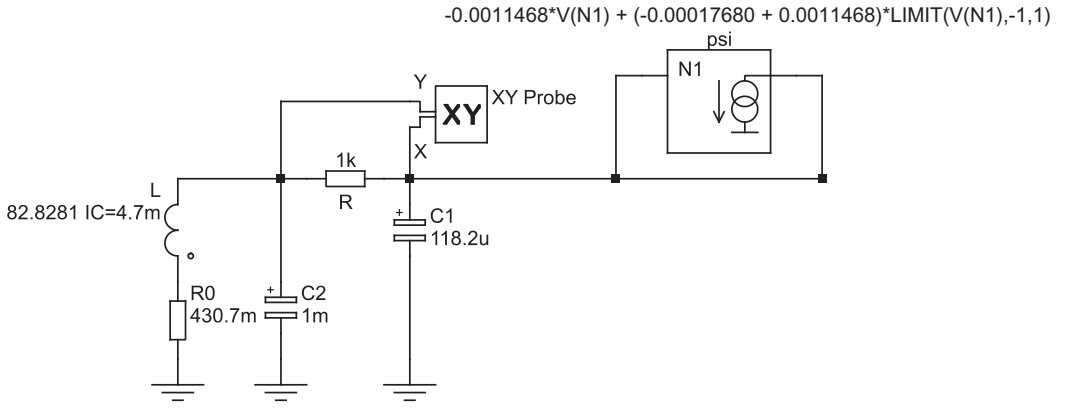

Figure 3. Chua circuit in SIMetrix SPICE.

2. Visualization of hidden Chua attractor in SPICE

Nowadays various Simulation Programs with Integrated Circuit Emphasis (SPICE) are widely used to analyze and design analog circuits [33]. Consider simulation of hidden Chua attractor in SIMetrix SPICE[1] for the following parameters: $C_1 = 118.2u$, $C_2 = 1m$, $L = 82.8281$, $R0 = 430.7m$, $R = 1k$, $G_b = -0.0001768$, $G_a = -0.0011468$ ($\alpha = 8.4562$, $\beta = 12.0732, \gamma = 0.0052, m_0 = -0.1768, m_1 = -1.1468$).

```
1  *#SIMETRIX
2  R1 L_P 0 430m
3  X$psi psi_OUT psi_OUT $$arbsourcepsi pinnames: N1 OUT
4  .subckt $$arbsourcepsi N1 OUT
5  B1 OUT 0 I=-0.0011468*V(N1) +  (-0.00017680+0.0011468)*LIMIT(V(N1),-1,1)
6  .ends
7  C1 psi_OUT 0 118.2u IC=2  BRANCH={IF(ANALYSIS=2,1,0)}
8  C2 C2_P 0 1m IC=1  BRANCH={IF(ANALYSIS=2,1,0)}
9  L L_P C2_P 82.8281 IC=-4m  BRANCH={IF(ANALYSIS=2,0,1)}
10 R psi_OUT C2_P 1k
11 .graph "XY(C2_P, psi_OUT)"  initxlims=false
12 .TRAN 0 50 0 10m UIC
```

Corresponding SPICE realization of Chua circuit (see Figure 1) is shown in Figure 3. Here $L, R0, R, C1, C2$ correspond to the elements of Chua circuit, and element *XY Probe* is used to measure the capacitor voltages (and to plot projection of trajectories on (v_1, v_2)-plane). The Chua diode with characteristic $f(v_1)$ is realized as 'Arbitrary Source' (voltage-controlled current source) **psi**.

For the considered values of parameters there are three equilibria in the system: the stable focus at the origin and two symmetric saddle equilibria $S_{\pm}$. To check that an attractor is hidden, we have to demonstrate that the trajectories from certain small vicinities of equilibria are not attracted by the attractor. Figures 4 and 5 show SPICE simulation[2] of a trajectory in a vicinity of zero equilibrium and a trajectory with initial condition corresponding to the unstable manifold of the saddle (in both cases the considered trajectories are attracted to the zero equilibrium). Projection of twin symmetric chaotic attractors on the plane (v_1, v_2) and attraction to the zero equilibrium are shown in Figure 6.

Simulation self-excited Chua attractor in SPICE for the parameters $C_1 = 66.667u$, $C_2 = 1m$, $L = 35.7143$, $R0 = 0$, $R = 1k$, $G_b = -0.00071429$, $G_a = -0.0011468$ is shown in Figure 7. In this case the trajectory with initial data in a vicinity of zero equilibrium is attracted by the self-excited attractor.

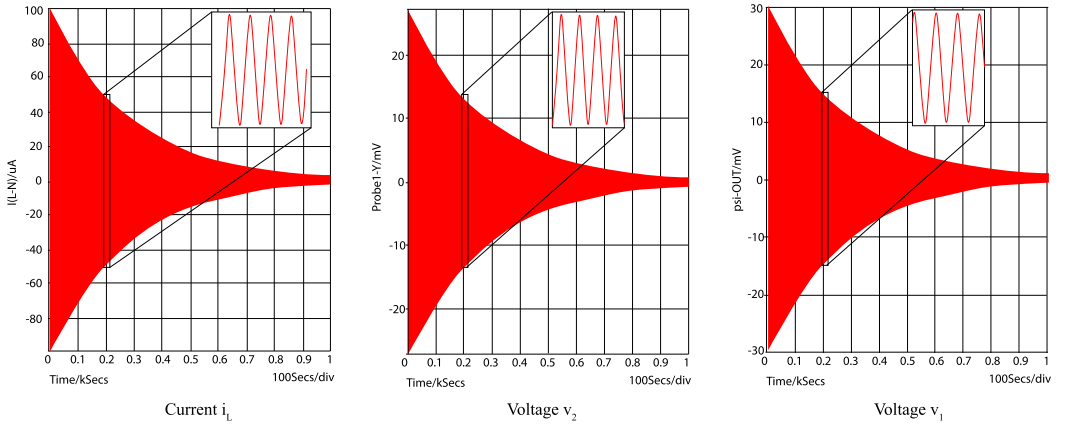

Figure 4. Vicinity of zero equilibrium: $v_2(0) = 0, \quad v_1(0) = 0, \quad i_L(0) = 100$ u.

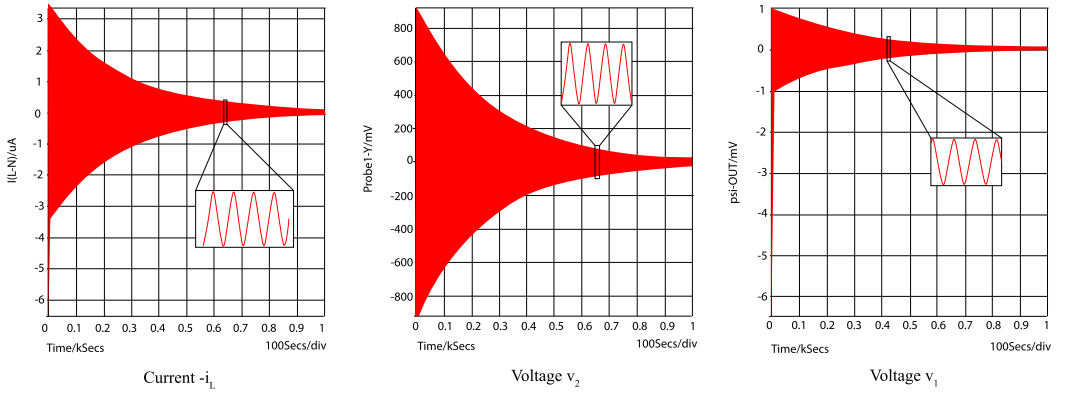

Figure 5. Vicinity of saddle point: $v_2(0) = -2.80804$ m, $\quad v_1(0) = -6.5224, \quad i_L(0) = 6.5196$ m.

3. Synchronization of Chua circuits

Nowadays various chaotic secure communication systems based on Chua circuits and other electronic generators of chaotic oscillations are of interest [34–39]. The operation of such systems is based on the synchronization chaotic signals of two chaotic identical generators (transmitter and receiver) for different initial data. The control signal proportional to the difference between the circuits signals, adjusts the state of the receiver. The multistability and existence of hidden attractors may lead to improper operation of such systems.

Consider now two x-coupled Chua systems

$$\begin{aligned}
\frac{dx}{dt} &= \alpha(y - x - \psi(x) + \delta(\tilde{x} - x)),\\
\frac{dy}{dt} &= x - y + z,\\
\frac{dz}{dt} &= -(\beta y + \gamma z),\\
\frac{d\tilde{x}}{dt} &= \alpha(\tilde{y} - \tilde{x} - \psi(\tilde{x}) + \delta(x - \tilde{x})),
\end{aligned}$$

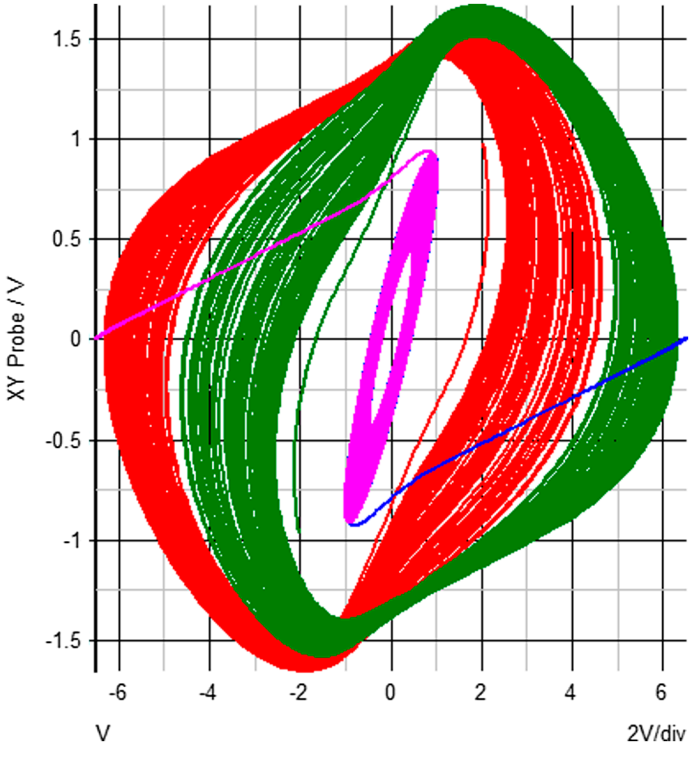

Figure 6. Voltage v_1 vs voltage v_2. Initial conditions for trajectories attracted to the stable zero equilibrium (blue and pink): pink — $v_2(0) = -2.80804$ m, $v_1(0) = -6.5224$, $i_L = 6.5196$ m; blue — $v_2(0) = 2.80804$ m, $v_1(0) = 6.5224$, $i_L = -6.5196$ m. Initial condition for red trajectory (visualizing a hidden attractor – projection on the plane): $v_2(0) = 1, v_1(0) = 2, i_L = -4$ m. Initial condition for green trajectory (visualizing symmetric hidden attractor – projection on the plane): $v_2(0) = -1, v_1(0) = -2, i_L = 4$ m.

$$\begin{aligned} \frac{d\tilde{y}}{dt} &= \tilde{x} - \tilde{y} + \tilde{z}, \\ \frac{d\tilde{z}}{dt} &= -(\beta\tilde{y} + \gamma\tilde{z}), \end{aligned} \tag{4}$$

where δ is a coupling factor ($\delta = \frac{R}{R_{\text{coupling}}}$). The corresponding circuit is on Figure 8.

SIMetrix SPICE realization of the coupled Chua circuits is shown in Figure 9.

Here the passive elements of the first circuit: $L, R0, R, C1, C2$, are identical to the corresponding elements of the second circuit: $L1, R1, R2, C3, C4$. Elements *XYProbe* are used to measure capacitor voltages and plot them on (v_1, v_2)-plane. To simulate nonlinear elements with characteristic $f(v_1)$, we use SPICE elements 'Arbitrary Source' (voltage-controlled current source) **psi** and **psi1**. Resistor $R3$ connects two circuits and characterizes the distance between transmitter and receiver. Below it is shown that critical coupling value (i.e. maximum value for which synchronization takes place) is different for different choices of initial data because of the multistability and hidden attractors. In our experiments the simulation time is 100 seconds and minimal value of $R3$ is 1000.

Example 1: Consider initial data of the circuits on one of the symmetrical hidden attractors: Chua circuit 1 — $v_1(0) = -3.7727, v_2(0) = -1.3511, i_L(0) = 0.0047$; Chua circuit 2 — $v_1(0) = -3.6, v_2(0) = -1.3511, i_L(0) = 0.0047$ (see Figure 10).

Example 2: Consider initial data of the circuits on two symmetrical hidden attractors: Chua circuit 1: $v_1(0) = -3.7727, v_2(0) = -1.3511, i_L(0) = 0.0047$; Chua circuit 2: $v_1(0) = 3.7727, v_2(0) = 1.3511, i_L(0) = -0.0047$ (see Figure 11).

Figure 7. Voltage v_1 vs voltage v_2. Initial condition for red trajectory (visualizing a self-excited attractor-projection on the plane): $v_2(0) = 10\,\text{m}, \quad v_1(0) = 0, \quad i_L = 0.$

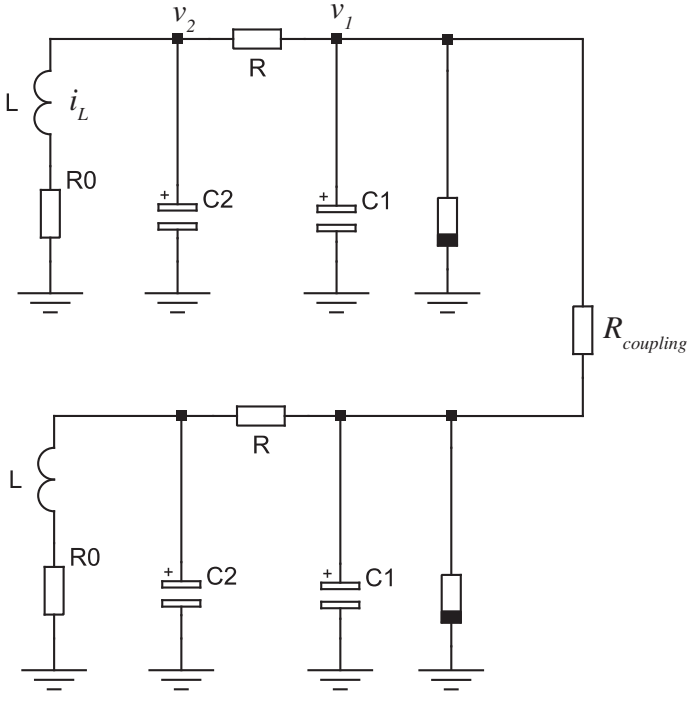

Figure 8. Two resistor-coupled Chua circuits.

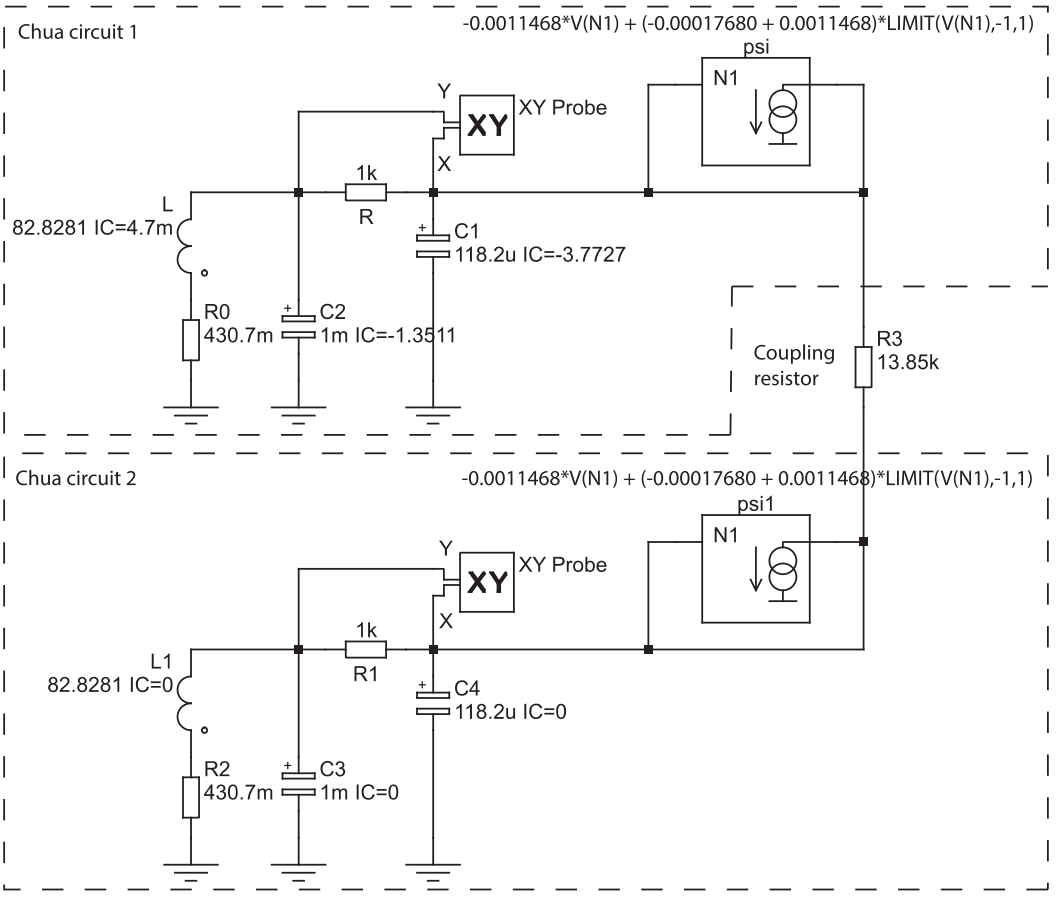

Figure 9. SIMetrix SPICE model of two coupled Chua's circuits.

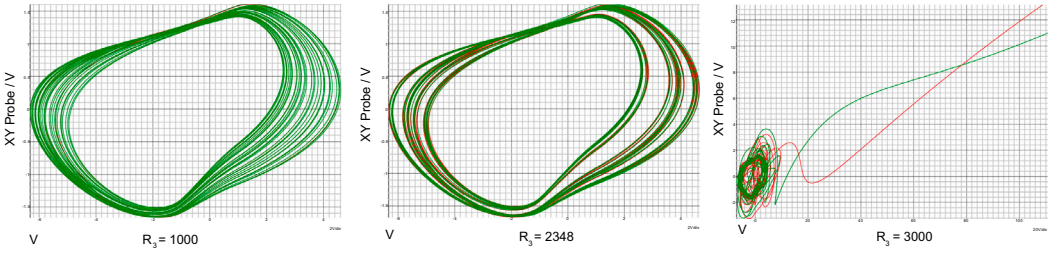

Figure 10. Example 1. Initial data of the circuits on the same hidden attractor. Critical coupling value $R3 = 2.348 \cdot 10^3$.

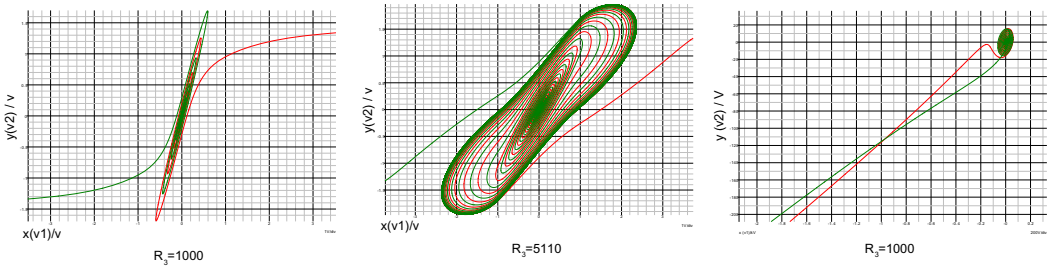

Figure 11. Example 2. Initial data of the circuits on two symmetrical hidden attractors. Critical coupling resistor value is $R3 = 5.110 \cdot 10^3$.

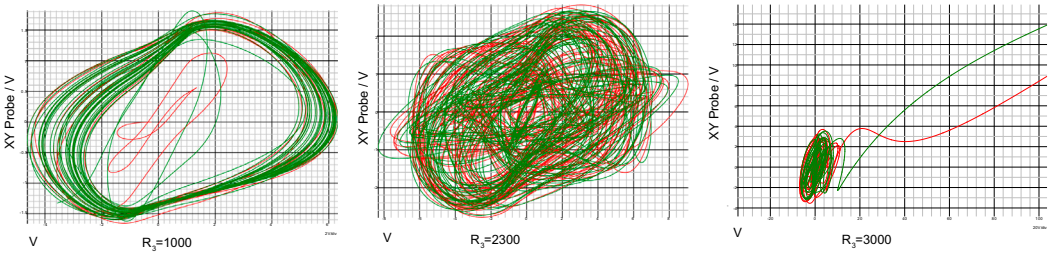

Figure 12. Example 3. Initial data of the circuits on a hidden attractor and stable zero equilibrium. Critical coupling resistor value is $R3 = 2.300 \cdot 10^3$.

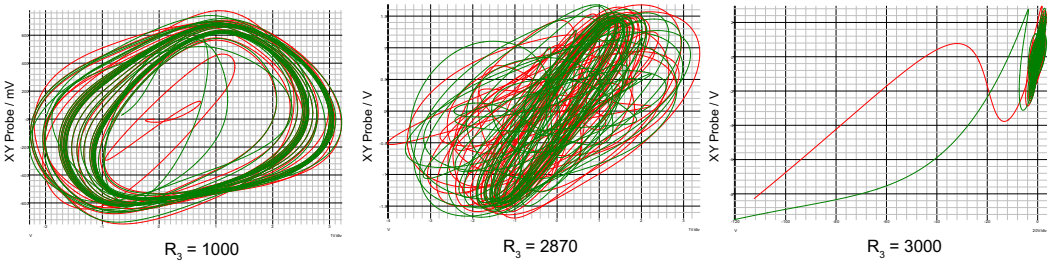

Figure 13. Example 4. Initial data of the circuits on a self-excited attractor and unstable zero equilibrium. Critical coupling resistor value is $R3 = 2.870 \cdot 10^3$.

Example 3: Consider initial data of the circuits on one of the symmetrical hidden attractors and stable zero equilibrium: Chua circuit 1 — $v_1(0) = -3.7727, v_2(0) = -1.3511, i_L(0) = 0.0047$; Chua circuit 2 — $v_1(0) = 0, v_2(0) = 0, i_L(0) = 0$ (see Figure 12).

Example 4: Consider the case of symmetrical self-excited attractors for the parameters $\alpha = 15$, $\beta = 28, \gamma = 0, m_0 = -5/7, m_1 = -8/7$ (corresponding SPICE parameters: $C_1 = 6.667 \cdot 10^{-5}$, $C_2 = 0.001$, $G_a = -7.1429 \cdot 10^{-4}$, $G_b = -1.1429$, $L = 35.7134$, $R_0 = 0$). Take the following initial data of the circuits on one of the symmetrical self-excited attractors and unstable zero equilibrium: Chua circuit 1 — $v_1(0) = -0.780, v_2(0) = -0.525, i_L(0) = 3.5$ m; Chua circuit 2 — $v_1(0) = 0, v_2(0) = 0, i_L(0) = 0$ (see Figure 13).

4. Conclusion

In conclusion we note that the simulation of circuit behavior by software, as well as the numerical integration of its dynamical model, is subject to numerical errors due to time discretization step (see, e.g. the corresponding examples with phase-locked-loop based circuits in [40–43]). Observation of hidden Chua attractors in physical experiments is discussed, e.g. in [23,44–47].

Notes

1. https://www.simetrix.co.uk/
2. In our experiment 'Max time step' is set to 10 m.

Disclosure statement

No potential conflict of interest was reported by the authors.

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